Notes on temperature-dependent lidar equations

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Outline*

- Temperature-dependent lidar equations
- Temperature-dependent functions for Howard University Raman Lidar (HURL)
- Errors estimates in Water Vapor Mixing Ratio and Aerosol Backscatter Ratio
- Theoretical consideration for WVMR
- Conclusions

* **Adam, M.**, Notes on temperature-dependent lidar equations, *J. Atmos. Oceanic Technol.*, **26**, 1021–1039 (2009)

Temperature-dependent lidar equations

Elastic lidar:

$$P(\lambda_L, r) = P_0 \frac{c\tau}{2} \frac{O_L(r) A \kappa(\lambda_L) \xi(\lambda_L) \left[F_L(T) N(r) \frac{d\sigma_t(\lambda_L, \pi)}{d\Omega} + \beta_{aer}(r) \right]}{r^2} \exp\left[-2 \int_0^r \alpha(\lambda_L, r') dr' \right]$$

Raman lidar:

$$P(\lambda_X, r) = P_0 \frac{c\tau}{2} \frac{O_X(r) A \kappa(\lambda_X) N_X(r) F_X(T) \frac{d\sigma_t(\lambda_X, \pi)}{d\Omega} \xi(\lambda_X)}{r^2} \exp\left\{ -\int_0^r \left[\alpha(\lambda_L, r') + \alpha(\lambda_X, r') \right] dr' \right\}$$

Temperature-dependent functions:

$$F_X(T) = \frac{\sum_i \frac{d\sigma(\lambda_{X,i}, T, \pi)}{d\Omega} \xi(\lambda_{X,i})}{\frac{d\sigma_t(\lambda_X, \pi)}{d\Omega} \xi(\lambda_X)}$$

$$F_L(T) = \frac{\sum_{n=N_2,O_2} \eta_n \sum_i \left(\frac{d\sigma(\lambda_{X,i}, T, \pi)}{d\Omega} \right)_n \xi(\lambda_{X,i})}{\sum_{n=N_2,O_2} \eta_n \left(\frac{d\sigma_t(\lambda_X, \pi)}{d\Omega} \right)_n \xi(\lambda_X)}$$

$\xi(\lambda_{X,i})$ - interference filter efficiency (λ dependent)

$\kappa(\lambda_X)$ - all other system efficiencies (λ independent)

Temperature-dependent lidar equations

Temperature-dependent formulation for Water vapor mixing ratio (WVMR) and Aerosol Backscatter Ratio (ABR)

$$WVMR = C_1 \kappa(\lambda_N, \lambda_H) \frac{P(\lambda_H, r)}{P(\lambda_N, r)} \frac{F_N(T) \frac{d\sigma_t(\lambda_N, \pi)}{d\Omega} \xi(\lambda_N)}{F_H(T) \frac{d\sigma_t(\lambda_H, \pi)}{d\Omega} \xi(\lambda_H)} \Delta\tau(\lambda_N, \lambda_H, r)$$

$$ABR = 1 - F_L(T) + C_2 \kappa(\lambda_N, \lambda_L) F_N(T) \frac{P(\lambda_L, r)}{P(\lambda_N, r)} \frac{\frac{d\sigma_t(\lambda_N, \pi)}{d\Omega} \xi(\lambda_N)}{\frac{d\sigma_t(\lambda_L, \pi)}{d\Omega} \xi(\lambda_L)} \Delta\tau(\lambda_N, \lambda_L, r)$$

$C_1 \cong 0.485$, $C2 \cong 0.78$

$$\Delta\tau(\lambda_N, \lambda_H, r) = \exp\left\{ -\int_0^r \left[\alpha(\lambda_N, r') - \alpha(\lambda_H, r') \right] dr' \right\}$$ - differential transmission

$$\kappa(\lambda_N, \lambda_X) = \frac{\kappa(\lambda_N)}{\kappa(\lambda_X)}$$ - calibration factor

Temperature-dependent functions for HURL

Rayleigh backscatter differential cross section

$$F_L(T) = \frac{\sum_{n=N_2,O_2} \eta_n \sum_i \left(\frac{d\sigma(\lambda_{X,i},T,\pi)}{d\Omega} \right)_n \xi(\lambda_{X,i})}{\sum_{n=N_2,O_2} \eta_n \left(\frac{d\sigma_t(\lambda_X,\pi)}{d\Omega} \right)_n \xi(\lambda_X)}$$

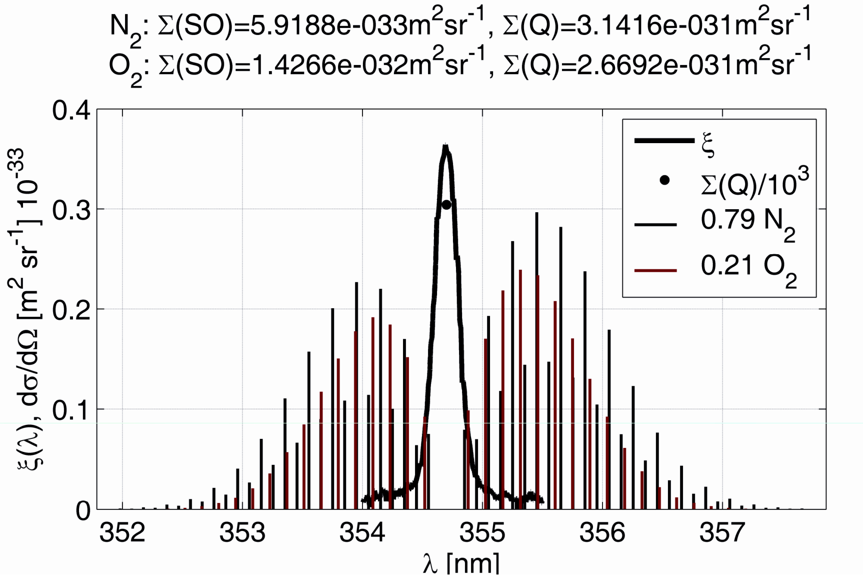
$$Q: \left(\frac{\partial\sigma}{\partial\Omega} \right)_J = \frac{112\pi^4}{45} (\nu_0)^4 \frac{g_{n,J}(2J+1)\exp\left(-\frac{E_{rot,J}}{K_B T} \right)}{Q_{rot}} \left(\frac{45}{7} a^2 + \frac{J(J+1)}{(2J-1)(2J+3)} \gamma^2 \right), J = 0,1,2,...,$$

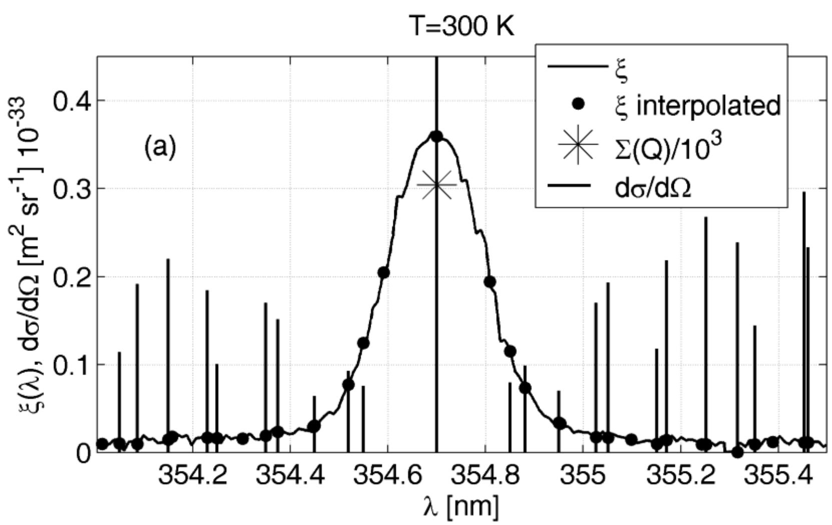
$$S: \left(\frac{\partial\sigma}{\partial\Omega} \right)_J = \frac{112\pi^4}{45} (\nu_0 - \Delta\nu)^4 \frac{g_{n,J}(2J+1)\exp\left(-\frac{E_{rot,J}}{K_B T} \right)}{Q_{rot}} \frac{3(J+1)(J+2)}{2(2J+1)(2J+3)} \gamma^2, J = 0,1,2,...,$$

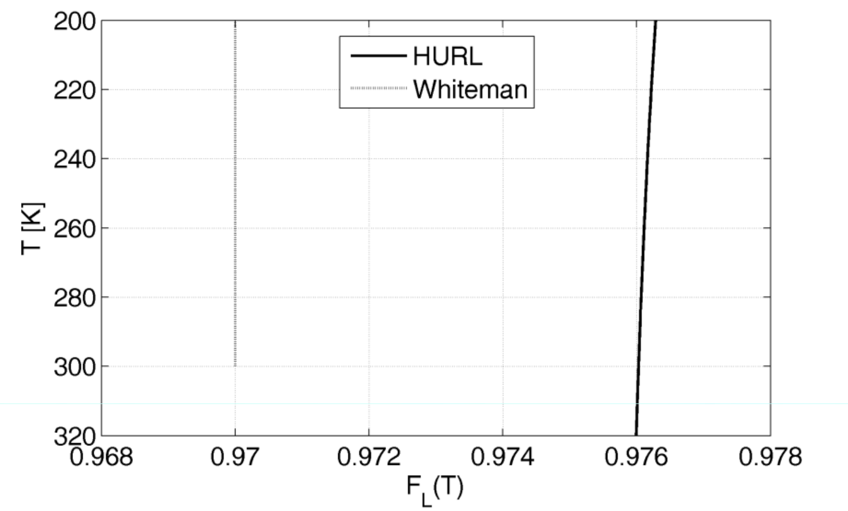
$$O: \left(\frac{\partial\sigma}{\partial\Omega} \right)_J = \frac{112\pi^4}{45} (\nu_0 - \Delta\nu)^4 \frac{g_{n,J}(2J+1)\exp\left(-\frac{E_{rot,J}}{K_B T} \right)}{Q_{rot}} \frac{3J(J-1)}{2(2J+1)(2J-1)} \gamma^2, J = 2,3,4,....$$

Temperature-dependent functions for HURL

Rayleigh backscatter differential cross section







Narrow filter: FWHM=0.25nm

Whiteman: theoretical Gaussian with FWHM=0.3nm

Nitrogen VR backscatter differential cross section

$$F_X(T)=\frac{\sum_i \frac{d\sigma\left(\lambda_{X,i},T,\pi\right)}{d\Omega}\xi\left(\lambda_{X,i}\right)}{\frac{d\sigma_t\left(\lambda_X,\pi\right)}{d\Omega}\xi\left(\lambda_X\right)}$$

$$Q:\left(\frac{\partial\sigma}{\partial\Omega}\right)_J=\frac{112\pi^4}{45}\left(\nu_0-\nu_{vib}+\Delta\nu\right)^4\frac{g_{n,J}\left(2J+1\right)\exp\left(-\frac{E_{rot,J}}{K_BT}\right)}{Q_{rot}}\frac{h}{8\pi^2c\nu_{vib}}\frac{1}{1-\exp\left(-\frac{hc\nu_{vib}}{K_BT}\right)}\times\left(\frac{45}{7}a'^2+\frac{J\left(J+1\right)}{\left(2J-1\right)\left(2J+3\right)}\gamma'^2\right),\ J=0,1,2...,$$

$$S:\left(\frac{\partial\sigma}{\partial\Omega}\right)_J=\frac{112\pi^4}{45}\left(\nu_0-\nu_{vib}+\Delta\nu\right)^4\frac{g_{n,J}\left(2J+1\right)\exp\left(-\frac{E_{rot,J}}{K_BT}\right)}{Q_{rot}}\frac{h}{8\pi^2c\nu_{vib}}\frac{1}{1-\exp\left(-\frac{hc\nu_{vib}}{K_BT}\right)}\times\frac{3\left(J+1\right)\left(J+2\right)}{2\left(2J+1\right)\left(2J+3\right)}\gamma'^2,\ J=0,1,2...,$$

$$O:\left(\frac{\partial\sigma}{\partial\Omega}\right)_J=\frac{112\pi^4}{45}\left(\nu_0-\nu_{vib}+\Delta\nu\right)^4\frac{g_{n,J}\left(2J+1\right)\exp\left(-\frac{E_{rot,J}}{K_BT}\right)}{Q_{rot}}\frac{h}{8\pi^2c\nu_{vib}}\frac{1}{1-\exp\left(-\frac{hc\nu_{vib}}{K_BT}\right)}\times\frac{3J\left(J-1\right)}{2\left(2J+1\right)\left(2J-1\right)}\gamma'^2,\ J=2,3,4,....$$

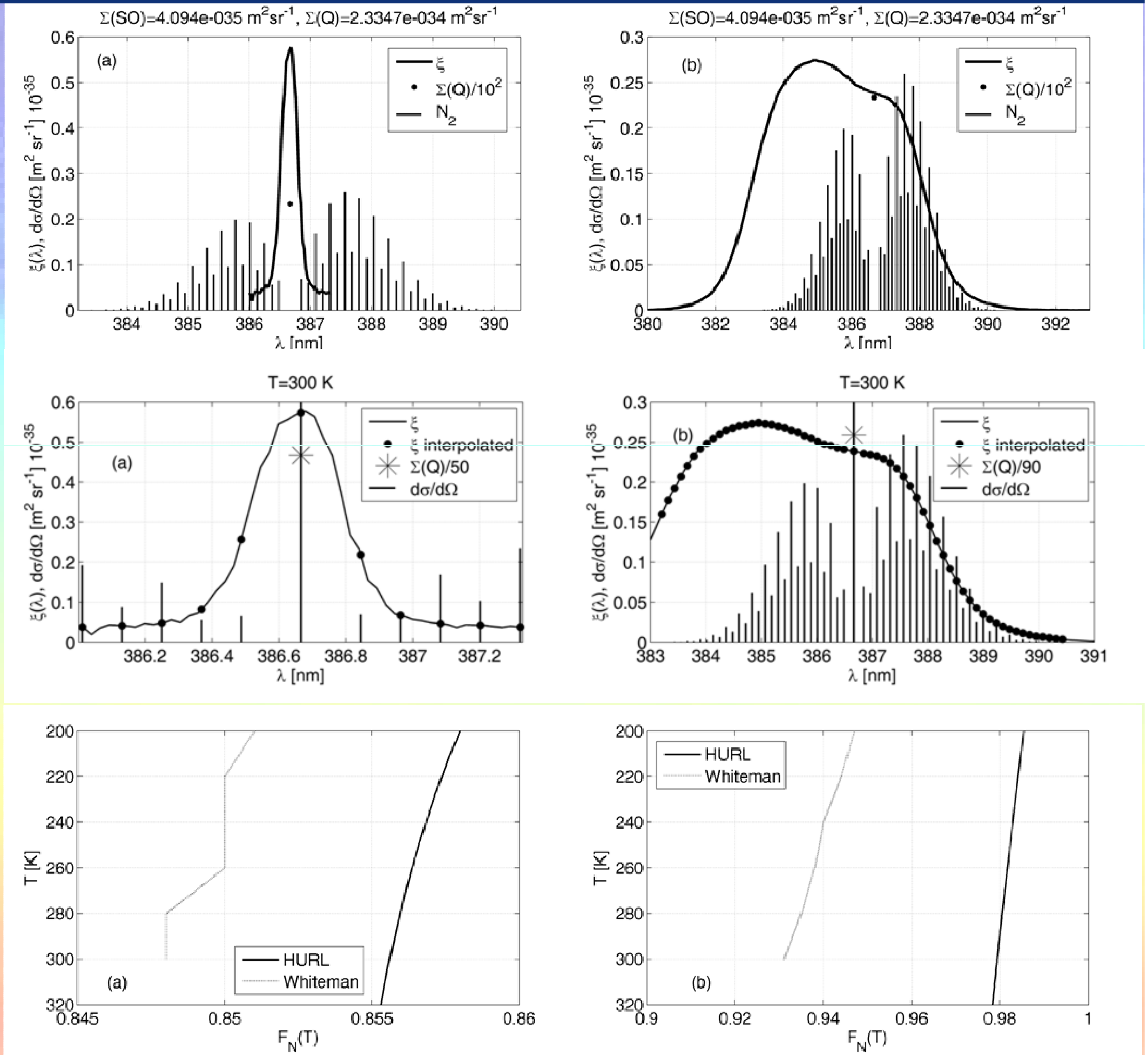
Temperature-dependent functions for HURL

Nitrogen VR backscatter differential cross section

Narrow filter:
FWHM=0.25nm

Wide filter:
FWHM=5nm

Whiteman: theoretical Gaussian with
FWHM=0.3nm
FWHM=2nm



Temperature-dependent functions for HURL

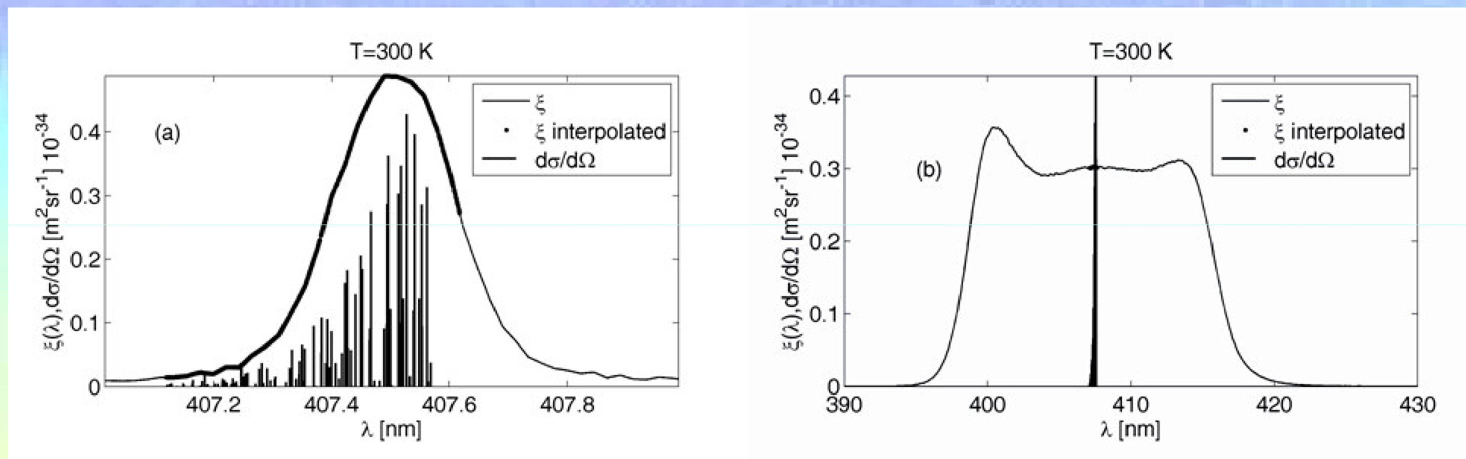
Water Vapor backscatter differential cross section

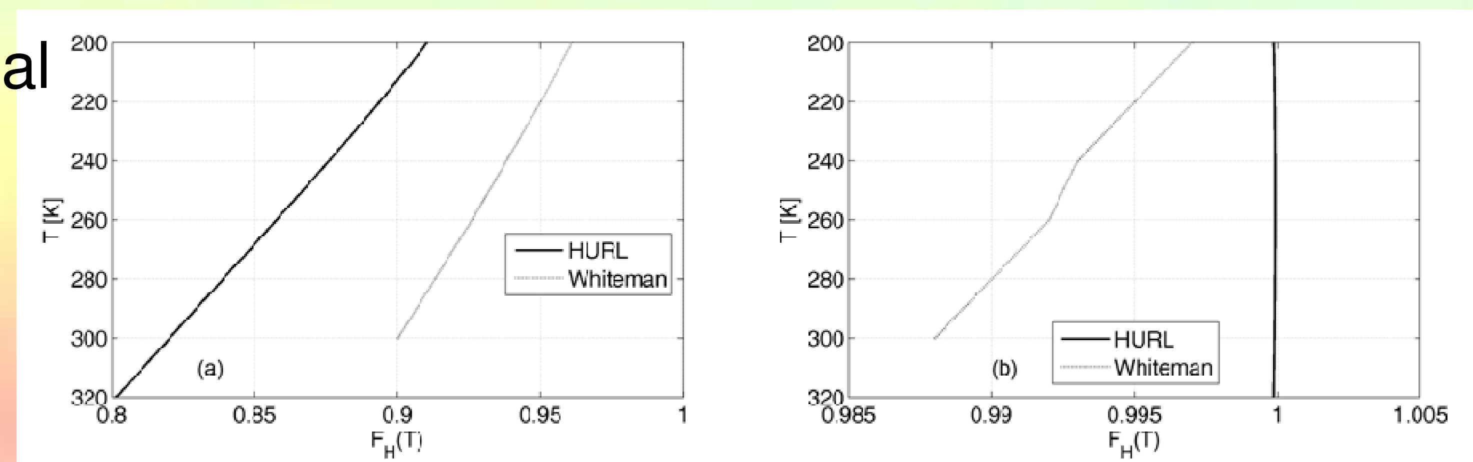
$$\left(\frac{\partial\sigma}{\partial\Omega}\right)_J = (\nu_0 - \nu_{vib} + \Delta\nu)^4 \frac{e^{-\frac{E_i}{K_B T}}}{Q(T)} \left(A^{XX} + A^{XY}\right)$$

Narrow filter:
FWHM=0.25nm

Wide filter:
FWHM=20nm

Whiteman: theoretical Gaussian with
FWHM=0.3nm
FWHM=2nm

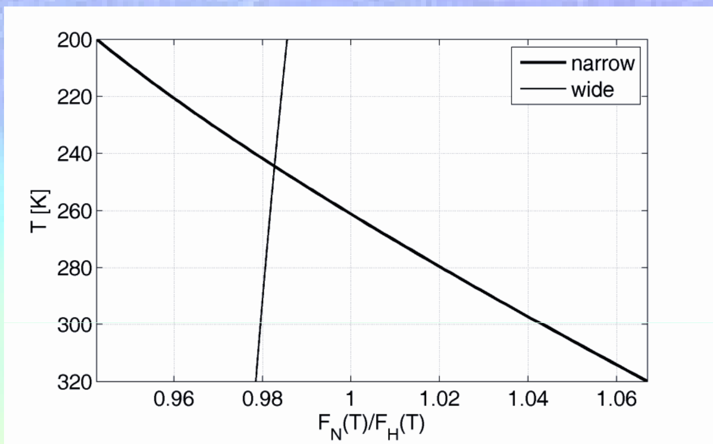


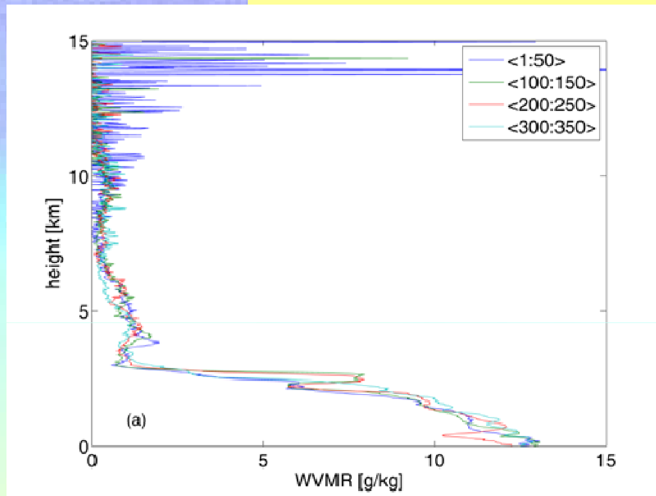


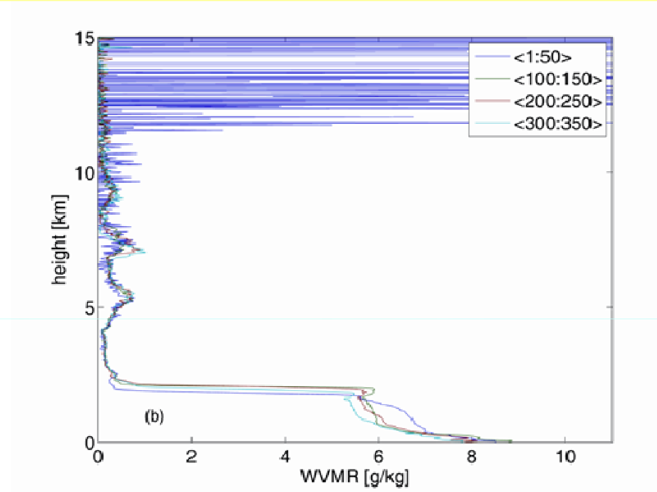
Errors estimates in WVMR and ABR

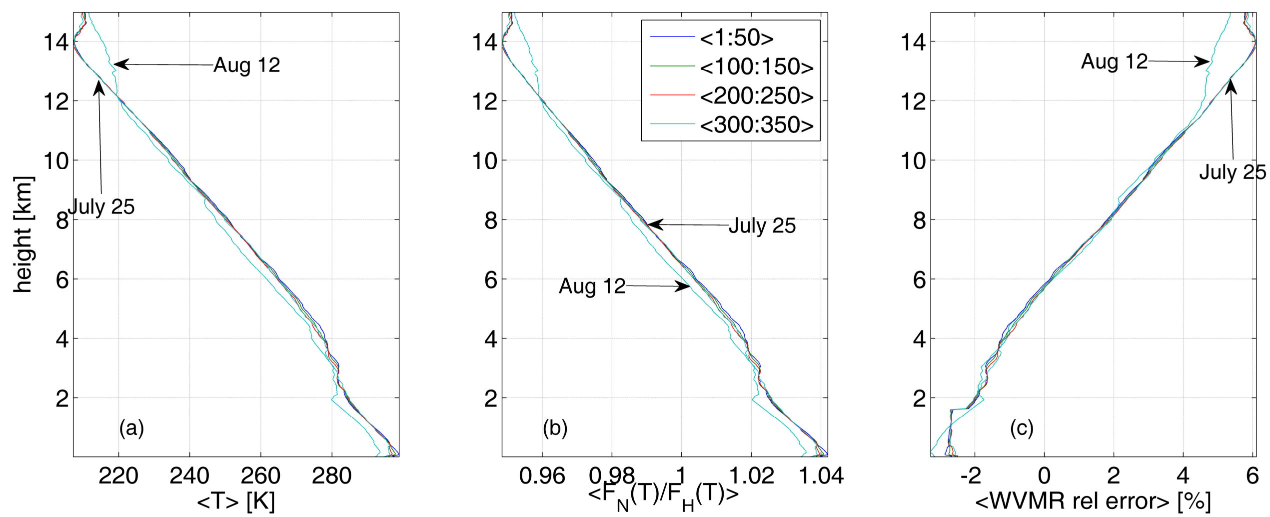
WVMR

$$WVMR = C_1 \kappa(\lambda_N, \lambda_H) \frac{P(\lambda_H, r)}{P(\lambda_N, r)} \frac{F_N(T) \frac{d\sigma_t(\lambda_N, \pi)}{d\Omega} \xi(\lambda_N)}{F_H(T) \frac{d\sigma_t(\lambda_H, \pi)}{d\Omega} \xi(\lambda_H)} \Delta\tau(\lambda_N, \lambda_H, r)$$





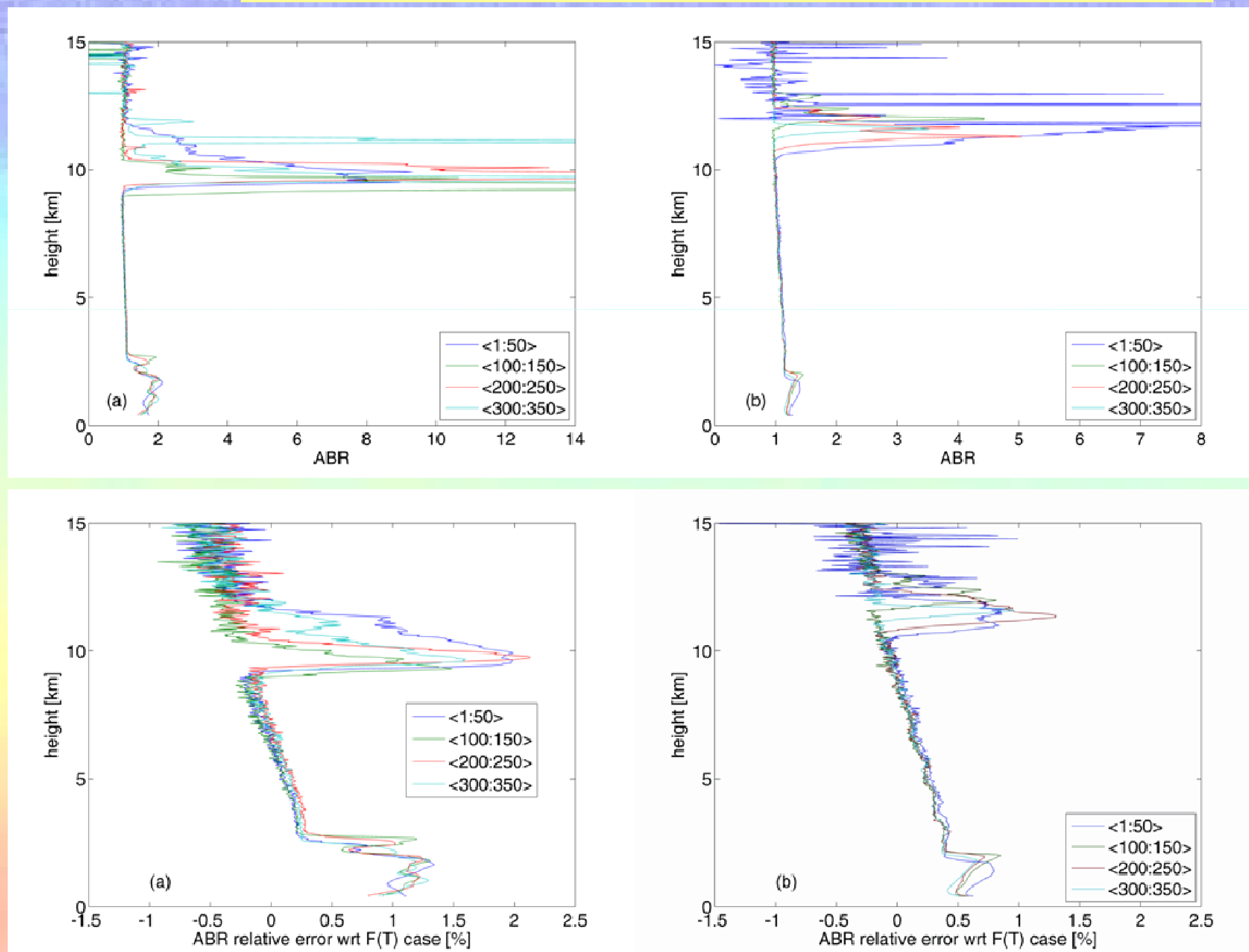




Errors estimates in WVMR and ABR

ABR

$$ABR = 1 - F_L(T) + C_2 \kappa(\lambda_N, \lambda_L) F_N(T) \frac{P(\lambda_L, r)}{P(\lambda_N, r)} \frac{\frac{d\sigma_t(\lambda_N, \pi)}{d\Omega} \xi(\lambda_N)}{\frac{d\sigma_t(\lambda_L, \pi)}{d\Omega} \xi(\lambda_L)} \Delta\tau(\lambda_N, \lambda_L, r)$$



Theoretical consideration for WVMR

$$WVMR = C_1 \kappa(\lambda_N, \lambda_H) \frac{P(\lambda_H, r)}{P(\lambda_N, r)} \frac{F_N(T) \frac{d\sigma_t(\lambda_N, \pi)}{d\Omega} \xi(\lambda_N)}{F_H(T) \frac{d\sigma_t(\lambda_H, \pi)}{d\Omega} \xi(\lambda_H)} \Delta\tau(\lambda_N, \lambda_H, r)$$

Goal: tilt filters such that $F_N(T)/F_H(T)$
- is closest to unity (smallest RMS)
- has the smallest variation (smallest RE)

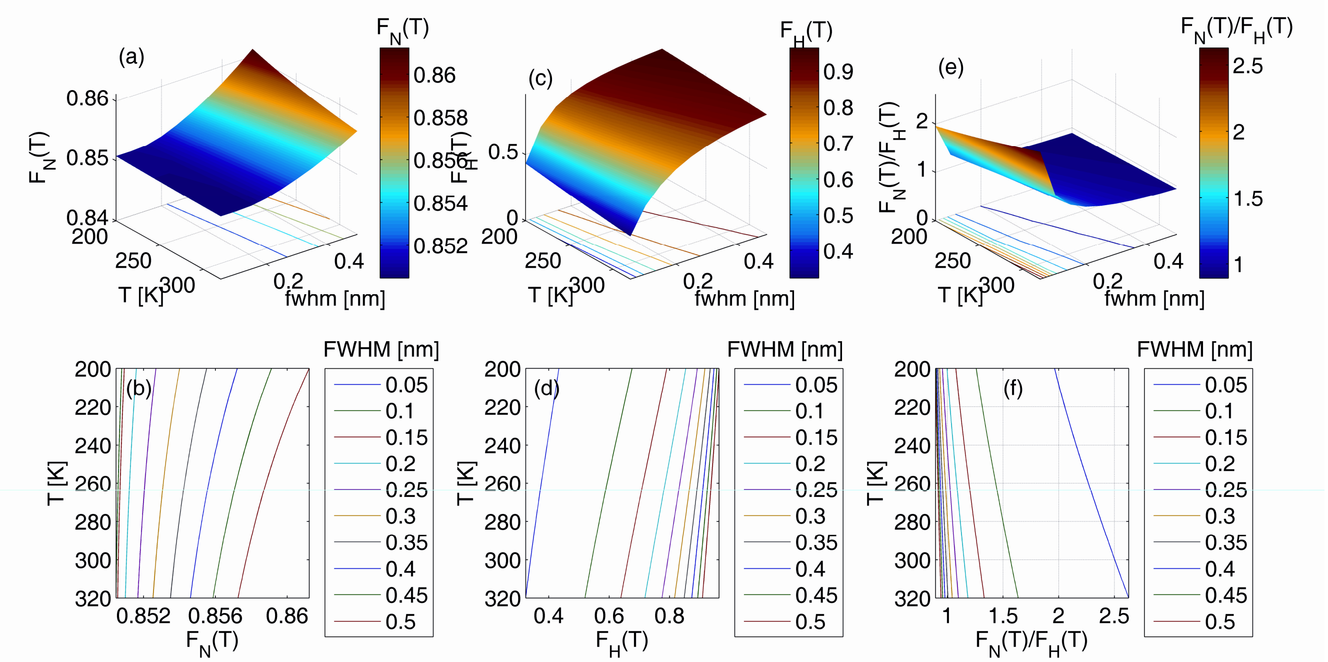
$$RMS[\%] = 100\sqrt{\frac{1}{n}\sum_{i=1}^{n}(F_i - 1)^2} \qquad RE[\%] = 100\frac{\sigma_{\langle F \rangle}}{\langle F \rangle}$$

Simulations: Gaussian filters (60% maximum efficiency) with FWHM from 0.05m to 0.5nm, T from 200 to 320K, and tilting from 0° to 3°. Normal incidence (no tilting): N_2 and water vapor filters are centered at λ=386.67 nm and λ=407.517 nm.

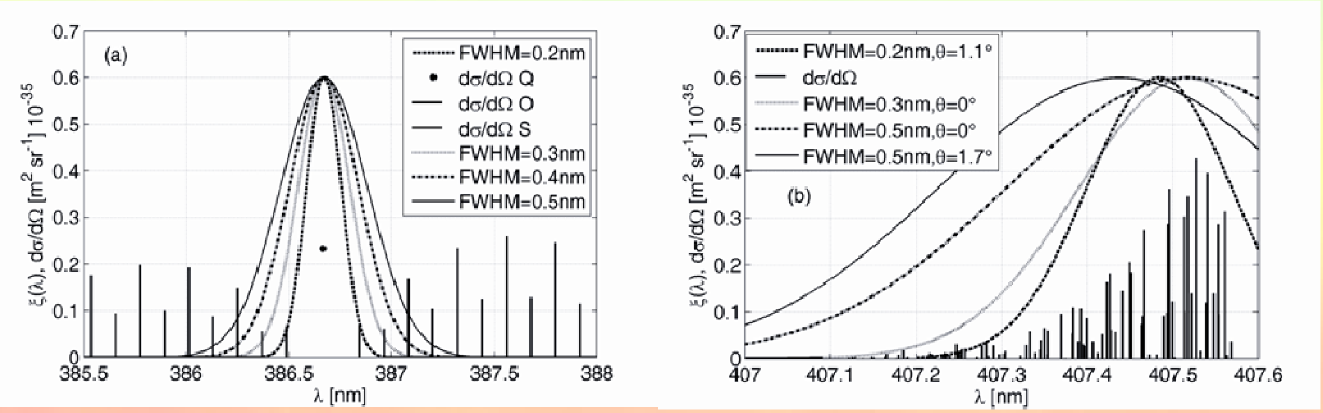
Theoretical consideration for WVMR

1) Identical filters, no tilting

smallest *RMS* (3.67 %) occurs for FWHM=0.3nm while the smallest *RE* (1.6 %) occurs at FWHM=0.5 nm



Maximum absolute deviation (*MAD*) for *RMS* is 6.88 % while *MAD* for *RE* is 2.91 %.

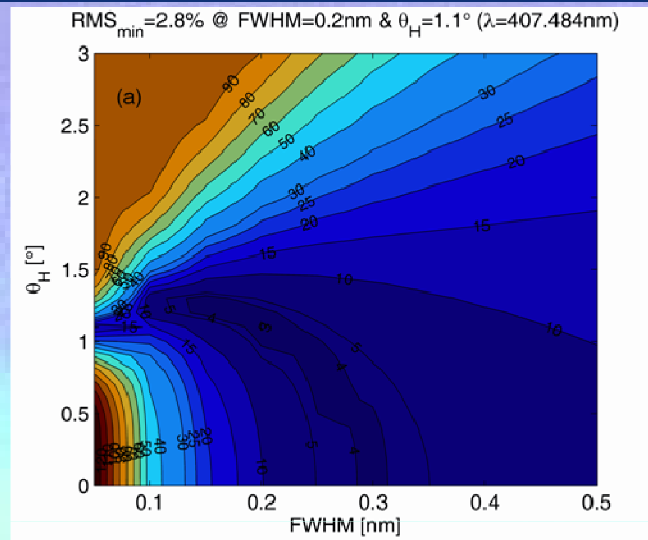


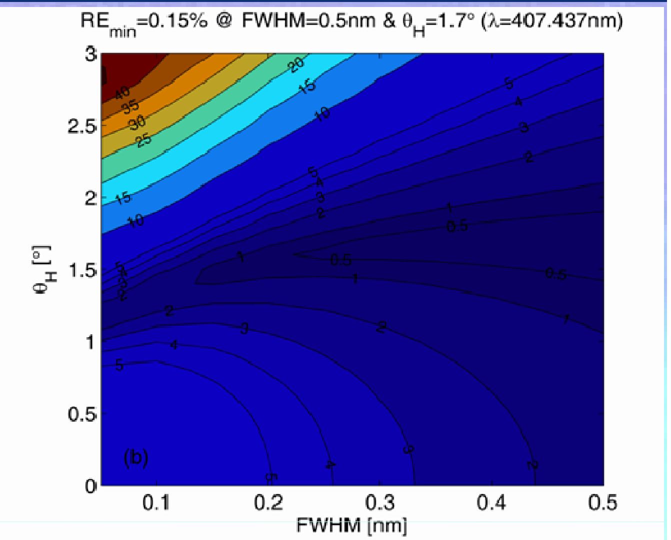
Theoretical consideration for WVMR

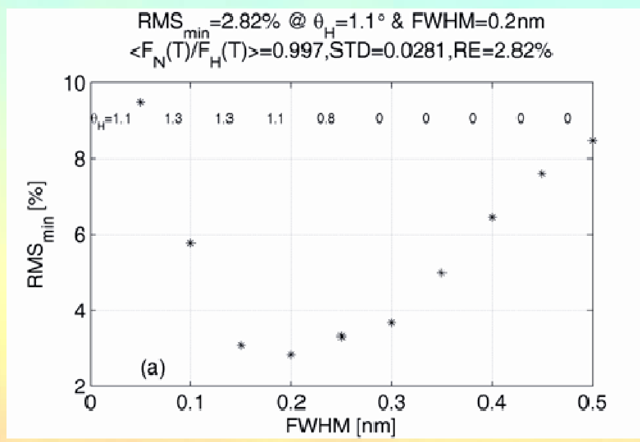
2) Identical filters, tilting only water vapor filter

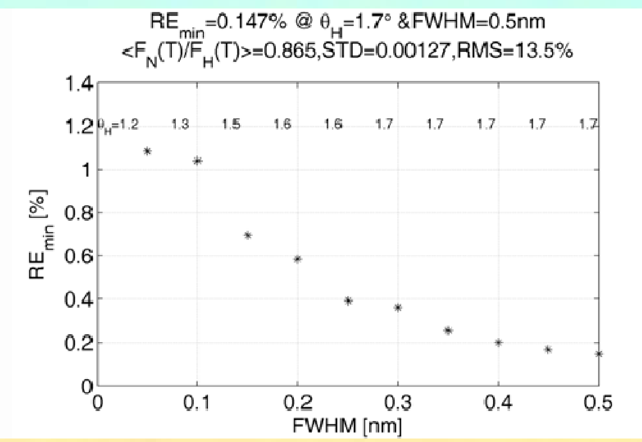
min(RMS_{min}) (2.82%) occurs at θ_H=1.1° and FWHM=0.2 nm

min(RE_{min}) (0.147%) occurs at θ_H=1.7° and FWHM=0.5 nm



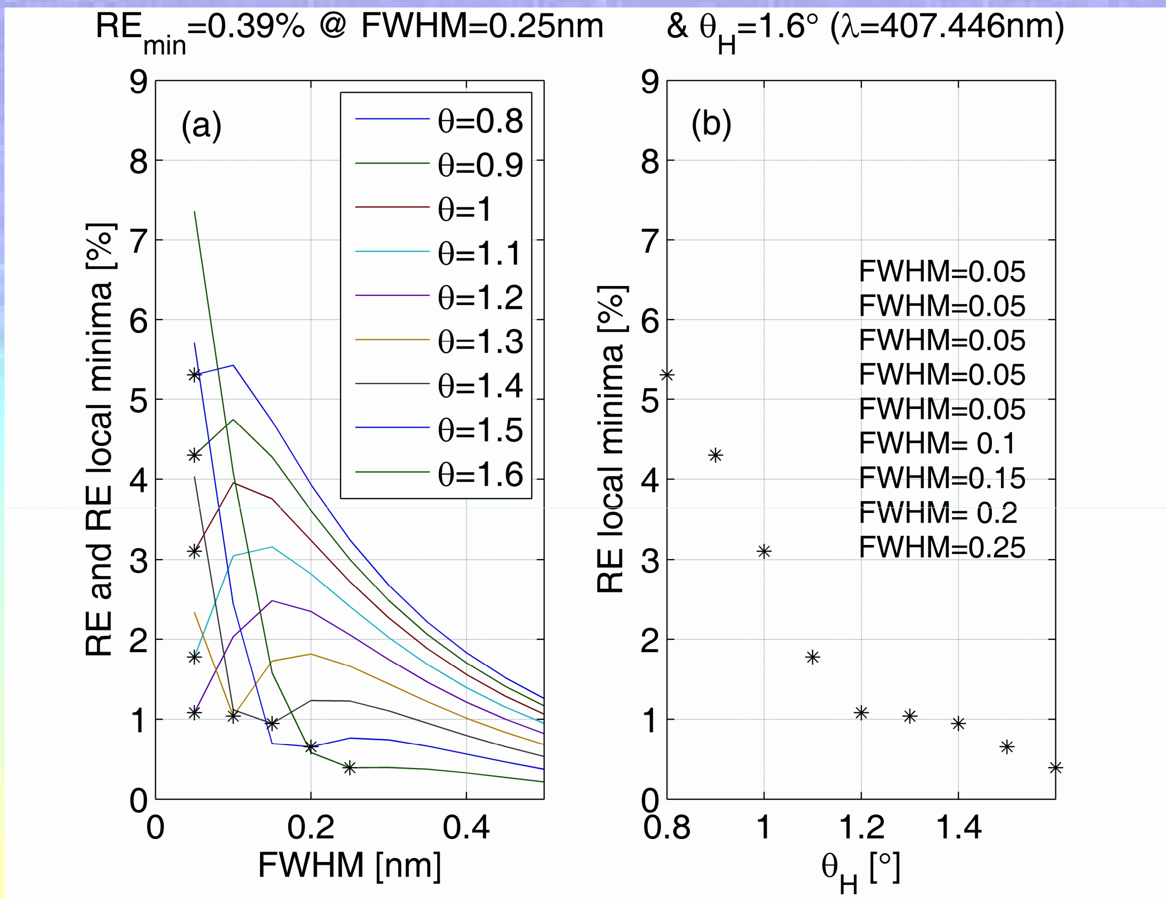






Theoretical consideration for WVMR

Which combination of FWHM and θ_H can still provide acceptable *RE* (within few percentages error)?



Search local minima:

- most of the local minima occur at the smallest FWHM (0.05 nm)

- the minimum of the local minima (0.39 %) occurs at θ_H=1.6° and FWHM=0.25 nm

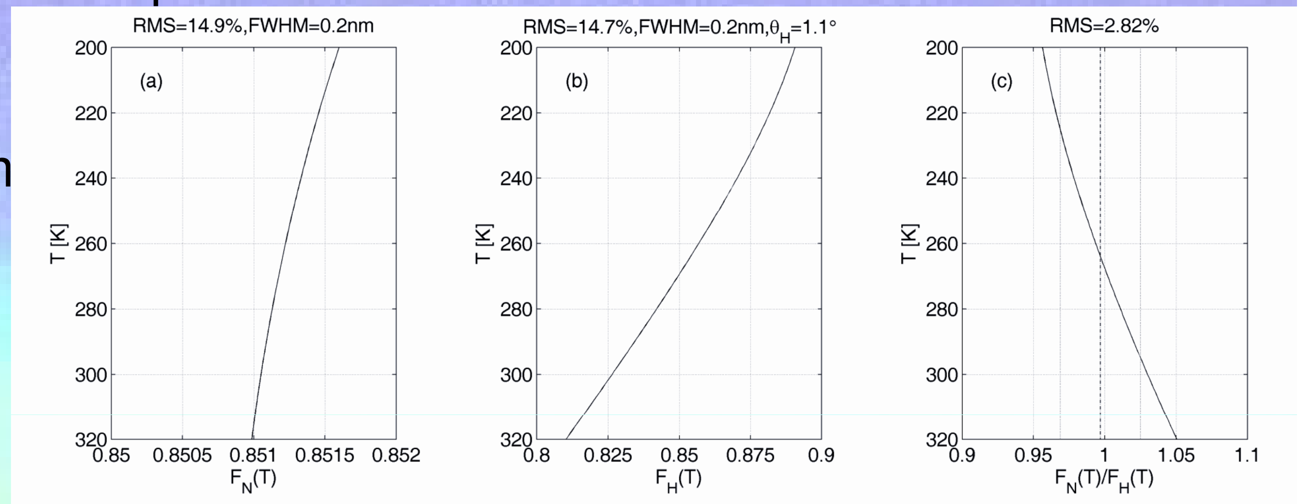
Thus, if want using smaller filters with *RE* within 1 % error ⇒

1) *RE*=0.95 % for FWHM=0.15 nm and θ_H=1.4°
2) *RE*=0.66 % for FWHM=0.20 nm and θ_H=1.5°
3) *RE*=0.39 % for FWHM=0.25 nm and θ_H=1.6°

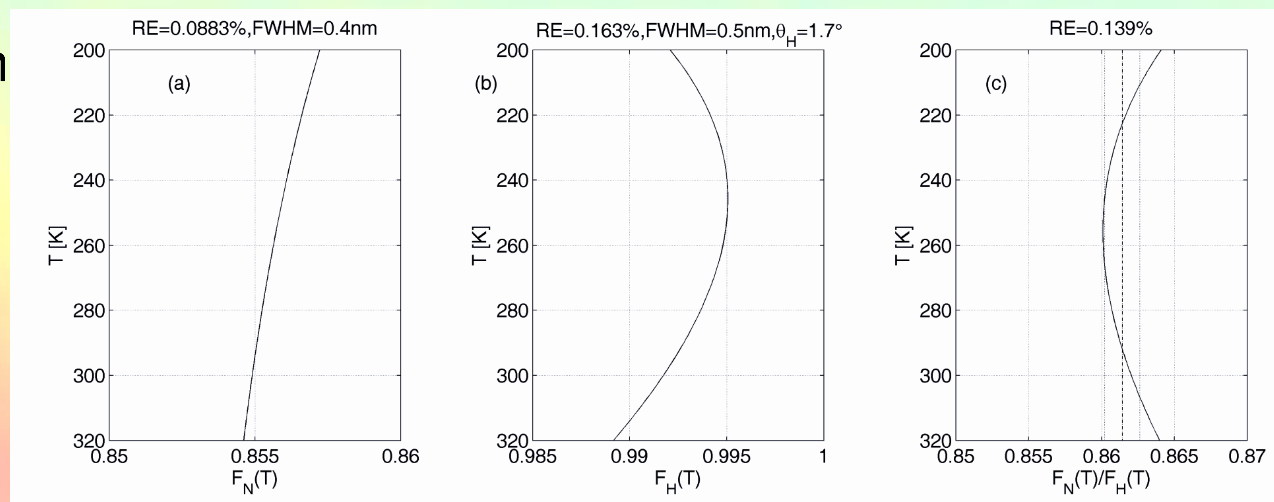
Theoretical consideration for WVMR

3) Different filters, tilting only water vapor filter

- the smallest *RMS* (2.82 %) for a N_2 filter with FWHM=0.2nm and for a water vapor filter with FWHM=0.2nm and θ_H=1.1°
- *MAD* of 5 % at *T*=320 K
- errors larger than 2 % for *T*>290 K and *T*< 242 K



- the smallest *RE* (0.139 %) for a N_2 filter with FWHM=0.4nm and for a water vapor filter with FWHM=0.5nm and θ_H=1.7°
- *MAD* ~ 0.3 % at *T*=320 K and *T*=200 K
- errors larger than 0.2 % occur for *T*>303 K and *T*<214 K



Conclusions

- For the experimental data analyzed:
  - errors in evaluating *WVMR* were up to ~ 6 %
  - errors in evaluating *ABR*, the errors were up to ~ 1.3 % the PBL and up to ~ 2.2 % within Cirrus clouds regions
- theoretical computations reveal:
  - the best combination of the filters with the optimum tilting for water vapor filter give a *RMS* ~3%, with a maximum absolute deviation of ~7 % (not suitable when computing the WVMR)
  - for small *RE* (profile almost constant) several combinations of the filters can be found such that *RE* is smaller than 1 % (minimum *RE* is found as 0.14 % while the maximum absolute deviation is 0.30 %)
  - easily *RMS* or *RE* computation for a specific combination of filters

THANK YOU!

QUESTIONS?